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Viscous dissipation effects in microtubes and microchannels

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Abstract

The effects of viscous dissipation on the temperature field and ultimately on the friction factor have been investigated using dimensional analysis and experimentally validated computer simulations. Three common working fluids, i.e., water, methanol and iso-propanol, in different conduit geometries were considered. It turns out that for microconduits, viscous dissipation is a strong function of the channel aspect ratio, Reynolds number, Eckert number, Prandtl number and conduit hydraulic diameter. Thus, ignoring viscous dissipation could affect accurate flow simulations and measurements in microconduits.

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1. Introduction

Except for very viscous fluids at relatively high speed, viscous dissipation effects are typically ignored in macroconduits. In contrast, even for common fluids at laminar Reynolds numbers, frictional effects in microsize conduits may change the temperature fields measurably. As a result, the local viscosity, friction factor, flow structure and scalar transport variables may be affected. Applications include microheat exchangers for cooling electronic systems [1] and valveless pumping mechanisms [2].

Unfortunately, experimental observations of fluid flow and heat transfer are often conflicting as reported by Papautsky et al. [3] and Koo and Kleinstreuer [4]. Papautsky et al. [3] reviewed previous experimental results and discussed the flow of gas, water, alcohol and silicon oil in microconduits. They emphasized the necessity for more experiments to draw conclusions regarding microscale effects. Koo and Kleinstreuer [4] analyzed experimental results for liquid flow in microchannels and simulated numerically the effect of surface roughness, inlet geometry and fluid temperature on the friction factor in order to establish key sources explaining the variance in experimental measurements.

For example, when the water temperature rises from 300 to 310 K, say, due to viscous dissipation, the kinematic-viscosity decreases by 20% which results in a 25% increase of the local Reynolds number. Thus, fluid temperature changes in microchannels may affect the friction factor and hence all transport phenomena [5]. Toh et al. [6] investigated numerically transport phenomena in heated microchannels. As expected, they found that heat input lowers the frictional losses, particularly at low Reynolds numbers. Judy et al. [7] observed measurable liquid flow temperature rises (e.g., 6.2 K for iso-propanol in a long square fused-silica channel of 74.1 μ m diameter for $Re \approx 300$) and related this to viscous dissipation. They suggested that the viscosity change due to temperature changes should be taken into account to estimate the friction factor. Pfhaler et al. [8] attributed viscosity changes to the channel size, and defined a viscosity ratio as $C^* = \frac{\mu_{\text{exit}}}{\mu_{\text{inlet}}}$, or $\frac{\mu_{\text{mean}}}{\mu_{\text{inlet}}}$, a parameter used in this study as well.

Tunc and Bayazitoglu [9] investigated the viscous dissipation effect in rarefied gas flow by varying the

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Nomenclature

A	area [m ²]
Br	Brinkman number $(=Ec \cdot Pr)$
С	viscosity ratio, $\frac{\mu_{\text{exit}}}{\mu_{\text{intra}}}$, or $\frac{\mu_{\text{mean}}}{\mu_{\text{intra}}}$
C_p	specific heat at constant pressure [J/kgK]
D	diameter [m]
Ec	Eckert number $\left(=\frac{\frac{1}{2}U_{ave}^2}{C_e\Lambda T}\right)$
f	friction factor
g	gravitational acceleration [ms ⁻²]
h	head loss [m]
k	thermal conductivity [Wm ⁻¹]
L	axial system length [m]
ln	natural logarithmic function
р	pressure [N m ⁻²]
Pr	Prandtl number $\left(=\frac{v}{\alpha}\right)$
q	heat flux [W m ⁻²]
r	radial coordinate [m]
R	pipe radius [m]
Re	Reynolds number $\left(=\frac{\rho U_{ave}D_{h}}{\mu}\right)$
Т	temperature [K]
u, U	axial velocities [m/s]
v	radial or y-directional velocity [m/s]
<i>x</i> , x	axial coordinates [m]
<i>y</i> , y, <i>z</i> , z	coordinates [m]

Greek symbols

α	thermal diffusivity $[m^2 s^{-1}]$				
и	dynamic viscosity $[kg m s^{-1}]$				
v	kinematic viscosity $[m^2 s^{-1}]$				
0	density [kg m ⁻³]				
τ	stress $[N m^{-2}]$				
Φ	viscous dissipation function $[s^{-2}]$				
1					
Supersci	ripts				
*	non-dimensional variables				
·	time rate				
Subscrit	Subscripts				
0	standard				
ave	averaged variable				
D	diameter-based				
exit	channel or tube exit				
f	frictional				
h	hydraulic				
inlet	channel or tube inlet				
m	mean				
r	radial direction				
w	wall				
vv	VY CL11				

Brinkman number $(Br = \frac{\text{viscous dissipation term}}{\text{thermal diffusion term}})$ in their nondimensionalized governing equations, and concluded that it plays an important role in heat transfer. Tso and Mahulikar [10–12] published three papers regarding the Brinkman number effect on convective heat transfer and flow transition. They stated that "the problem of heat dissipation accompanied by reducing the future size of IC chips cannot be solved by indefinitely reducing the microchannel dimensions without considering the effect of the Brinkman number, as this will lead to the same problem of viscous heat dissipation in the fluid which will offset the gains of high heat transfer coefficient associated with a reduction in channel size".

In this study, viscous dissipation effects on the evolution of temperature distributions are investigated, employing scale analyses and numerical solutions [13]. Such considerations are important in microfluidics in order to develop correct mathematical models for computer simulations. It should be noted that in all previous studies the viscous dissipation effect was just a minor part of the overall heat transfer process. Here, the importance of viscous dissipation is analyzed separately employing the experimental systems of Pfhaler et al. [8] and Judy et al. [7]. Three common working fluids, i.e., water, methanol and iso-propanol, as well as Reynolds number and channel geometry effects are considered.

2. Theory

Assuming steady laminar incompressible flow in tubes and channels, scale analyses of reduced transfer equations are performed and their numerical solutions are provided (Fig. 1).

2.1. Governing equations and scale analyses in tubes

2.1.1. Tubular flow

To assess the possible impact of temperature changes due to viscous dissipation on the velocity profile in Poiseuille-type flows, the x-momentum equation reads:

$$0 = -\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \mu \frac{\partial u}{\partial r} \right) \tag{1}$$

Non-dimensionalization of Eq. (1) and replacing the net pressure force term with the net shear force evaluated at the tube wall yields

$$\mu \frac{\partial^2 u^*}{\partial r^{*2}} + \left(\frac{\mu}{r^*} + \frac{\mathrm{d}\mu}{\mathrm{d}T}\frac{\partial T}{\partial r^*}\right)\frac{\partial u^*}{\partial r^*} - 2\left\{\mu \frac{\partial u^*}{\partial r^*}\right\}_{r^*=1} = 0 \qquad (2)$$

where $r^* = \frac{r}{R}$ and $u^* = \frac{u}{U_{ave}}$.

For a typical radial temperature difference of $0 \le \Delta T \le 20$ K in iso-propanol, which, of the three flu-



Fig. 1. System configurations for microchannel and microtube.



Fig. 2. Effects of viscosity changes for different radial temperature distributions ΔT on tubular velocity profiles.

ids, has a viscosity most sensitive to temperature changes, the velocity profiles based on Eq. (2) are shown in Fig. 2. Although the velocity change is measurable in the range $0 \le r^* \le 0.4$, the effect on the viscous dissipation term in the energy equation (see Eq. (3)) is minimal. Specifically, the impact on the viscous dissipation term (see Eq. (3)) is only 0.15% for $\Delta T = 10$ K and 0.61% for $\Delta T = 20$ K.

2.1.2. Tubular heat transfer

The energy equation can be expressed as:

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v_r \frac{\partial T}{\partial r} \right) = \frac{\partial}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(k_r r \frac{\partial T}{\partial r} \right) + \mu \Phi \pm \dot{q}$$
(3a)

where $\mu \Phi$ is the viscous dissipation term and \dot{q} is a possible heat source/sink. Specifically,

$$\mu \Phi = \mu \left\{ 2 \left[\left(\frac{\partial v_r}{\partial r} \right)^2 + \left(\frac{\partial u}{\partial x} \right)^2 \right] + \left(\frac{\partial v_r}{\partial x} + \frac{\partial u}{\partial r} \right)^2 \right\} \quad (3b)$$

Thus, with the stated assumption of hydrodynamically fully-developed flow, Eq. (3a) reduces to

$$\rho C_p \left(u \frac{\partial T}{\partial x} \right) = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right) + \mu \left(\frac{\partial u}{\partial r} \right)^2$$
(4)

where, $Re_{\rm D} = \frac{\rho U_{\rm mean} D}{\mu} = \frac{\rho U_{\rm max} R}{\mu}$. The axial velocity can be written as

J. Koo, C. Kleinstreuer | International Journal of Heat and Mass Transfer 47 (2004) 3159-3169

$$u(r) = \frac{Re_{\rm D}\mu}{\rho R} (1 - (r/R)^2)$$
(5)

The boundary conditions associated with Eq. (4) for a tube with, say, L/D = 800 are

$$T = 300 \text{ K} \text{ at } x = 0$$
 (6a)

$$\frac{\partial T}{\partial r} = 0$$
 at $r = 0$ (6b)

$$T = 300 \text{ K}$$
 at $r = R$
for constant temperature case (6c)

$$\frac{\partial T}{\partial r} = 0$$
 at $r = R$ for adiabatic case (6d)

$$\frac{\partial}{\partial x} \left(\frac{T - T_{\rm w}}{T_{\rm m} - T_{\rm w}} \right) = 0 \quad \text{at the exit} \tag{6e}$$

The material of microconduits, e.g., in MEMS devices, affects the type of thermal boundary condition. If the conduit is made of fused silica, as in the case reported by Judy et al. [7], the channel wall would work as an insulator due to the low heat conductivity of fused silica.

Scale analysis is performed to investigate the conduit size-effect on the relative magnitude of the viscous dissipation term with respect to other terms in the energy equation, assuming constant properties. Again, the axial diffusion term is negligible compared to radial diffusion. Hence, substituting Eq. (5) into Eq. (4) yields after non-dimensionalization with $T^* = \frac{T - T_w}{T_m - T_w}$, $r^* = r/R$, and $x^* = x/L$:

$$\frac{C_{\rho}Re_{\mathrm{D}}\mu}{RL}(1-r^{*2})\frac{\partial[T^{*}(T_{\mathrm{m}}(x)-T_{\mathrm{w}}(x))]}{\partial x^{*}} = \frac{k}{R^{2}r^{*}}(T_{\mathrm{m}}(x)-T_{\mathrm{w}}(x))\frac{\partial}{\partial r^{*}}\left(r^{*}\frac{\partial T^{*}}{\partial r^{*}}\right) + 4\frac{\mu^{3}Re_{\mathrm{D}}^{2}}{\rho^{2}}\frac{r^{*2}}{R^{4}}$$
(7)

The convective term should balance with the diffusion term and the viscous dissipation term. If the flow is thermally fully-developed for a constant heat flux boundary condition, then $T_{\rm m}(x) - T_{\rm w}(x) = \text{const. Com-}$ paring the two terms on the RHS, the diffusion term is inversely proportional to R^2 while the viscous dissipation term is inversely proportional to R^4 for a given fluid and Reynolds number. Thus, when the tube radius decreases, the viscous dissipation term increases much faster than the diffusion term. When ignoring the viscous dissipation term, there is a growing importance of the diffusion term $(\sim \frac{1}{R^2})$ in microtubes when compared to the convection term ($\sim \frac{1}{p}$). Furthermore, the Reynolds number effect on the viscous dissipation term is more important than on the convection term. While traditionally the Brinkman number is employed to describe the effect of viscous dissipation, we analyzed the temperature rise of a given working fluid as a function of the geometric ratio, $(\frac{L}{P})$, the Reynolds number, and the Eckert number. Thus Eq. (7) is rewritten as

$$\frac{(1-r^{*2})}{(T_{\rm m}(x)-T_{\rm w}(x))} \cdot \frac{\partial [T^*(T_{\rm m}(x)-T_{\rm w}(x))]}{\partial x^*} = \frac{1}{Pr \cdot Re_{\rm D}} \left(\frac{L}{R}\right) \frac{\partial}{\partial r^*} \left(r^* \frac{\partial T^*}{\partial r^*}\right) + 32 \frac{Ec}{Re_{\rm D}} \left(\frac{L}{R}\right) r^{*2} \qquad (8)$$

Clearly, for a given Pr and $\Delta T = (T_m - T_w)$, the Eckert number is proportional to the square of the Reynolds number. In turn, for a given set of fluid properties, the temperature rise is proportional to the Reynolds number. The impact of fluid temperature rise on the tube friction factor is as follows. By definition, $f \equiv \frac{8\tau_w}{\rho U_{ave}^2} = \frac{8\tau}{\rho U_{ave}^2} \frac{R}{r}$ and the friction head loss is $h_f \equiv f \frac{L}{D} \frac{U_{ave}^2}{2g}$; now, the pressure change in a tube can be written as $\Delta p = \frac{2\tau}{r} \Delta x = \rho g h_f = f \rho \frac{L}{D} \frac{U_{ave}^2}{2}$, where $\tau = \mu(T) \frac{\partial u}{\partial r}$. For an incompressible fluid, the density variation with temperature is negligible compared to the viscosity variation. Hence, the friction factor is proportional to the fluid viscosity which is a function of temperature only, and so the viscous dissipation effect on the friction factor can be analyzed by investigating variations in fluid temperature.

2.2. Numerical solutions for rectangular ducts

Two thermal flow cases for rectangular microchannels were considered in order to validate the computational approach and to assess the impact of the thermal entrance length on the fluid temperature development. In order to test the merits of a one-dimensional analysis, the effect of viscosity changes on the temperature rise was estimated for the cross-sectional mean temperature of iso-propanol flow. The energy equation was solved for the laboratory system of Judy et al. [7], i.e.,

$$\rho C_p U_{\text{ave}} A \frac{\mathrm{d}T_{\text{m}}}{\mathrm{d}x} = kA \frac{\mathrm{d}^2 T_{\text{m}}}{\mathrm{d}x^2} + \mu \int \int_A \Phi \,\mathrm{d}A \tag{9}$$

where A is the cross-sectional area of the channel. The data point [7] was obtained at the highest Reynolds number (Re = 300), so that the entrance effect should be the most significant compared to other cases considered in this study. The equation was solved using MAT-LAB®.

To investigate the thermal entrance effect, the full heat transfer equation was solved for hydraulically fullydeveloped flow and constant fluid properties except for the viscosity which was assumed to vary only with the (mean) flow temperature. Thus, Eq. (4) was discretized using a central difference scheme except for the convection term, which was discretized by using a first-order upwind scheme. The equation was integrated explicitly with the simple explicit method. The computational domain was refined until no mesh dependence of the results was observed. It took about 24 h to obtain a

3162

solution on a Linux box with dual Pentium 4, 2.0 GHz processors and 4 GB of memory.

2.3. Model validations

The computer simulation model was validated with experimental data sets of Judy et al. [7] and Pfhaler et al. [8]. In the case of Judy et al. [7], the flows of iso-propanol, methanol and water in a 74.1 µm square channel were simulated where the channel length was 1500 times the hydraulic diameter. The velocity profile and its gradient were calculated using the information given by White [14]. Because of the varying surface temperature, a mean wall temperature was used for T_w in Eq. (6e). The predicted iso-propanol temperature rise is close to the experimental data point of Judy et al. [7] as indicated in Fig. 3. Specifically, the dynamic viscosity of iso-propanol decreases about 20% per 10 K temperature change [15]. Hence, if the viscosity change due to viscous dissipation is neglected, then the Reynolds number will be underestimated by about 25%, which measurably affects the friction factor.

The thermal entrance length is much longer than the hydraulic entrance length due to the high Prandtl number (see Table 1). Fig. 3 shows that the thermal entrance for iso-propanol flow (Re = 300), is about half of the channel length. The thermal entrance behavior generates about a 1 K-difference at the exit compared to the non-thermal entrance case. However, for low Reynolds number flows, Re < 1, like those investigated by Pfhaler et al. [8], the thermal entrance behavior can be neglected. The experimental data of Pfhaler et al. [8] for iso-propanol flow in trapezoidal channels was compared with theoretical results obtained from a one-dimensional analysis. Fig. 4 shows the calculated temperature rise in the channels for various Reynolds numbers, which cover the experimental Reynolds number range. The actual trapezoidal channels ($D_{\rm h} \approx 16$, 30 and 40 µm) were simulated as rectangular conduits with the same hydraulic diameters and the same aspect ratios (i.e., average width/depth). Fig. 5 depicts comparisons between the theoretical results and the experimental data, where the latter were not interpreted in light of the viscous dissipation effects. In small and narrow channels,



Fig. 3. Working fluid effect on mean temperature rise (L/D = 1500, Re = 300).

Table 1 Mechanical and thermodynamic properties of various working fluids at 298 K [18,19]

	7 1.	0	L / J		
Working fluid	Density [kg/m ³]	Dynamic viscosity [Pa/s]	Thermal conductivity [W/m K]	Specific heat [J/kgK]	Pr
Water	996	8.67×10^{-4}	0.611	4178	5.9
Iso-propanol	779	1.95×10^{-3}	0.135	2606	37.6
Methanol	792	8.17×10^{-4}	0.200	2531	10.3



Fig. 4. Comparison of temperature rises in rectangular channels [8]. *Note*: the experimental trapezoidal channels of hydraulic diameter 30 and 40 µm were modeled as rectangular channels of the same hydraulic diameters and the same aspect ratio.



Fig. 5. Comparison between experimental data [8] and computational results.

the dimensionless viscosity parameter, $C^* = \frac{\mu_{\text{exit}}}{\mu_{\text{inlet}}}$, decreases rapidly as the Reynolds number increases even in the very low Reynolds number range (i.e., $Re \ll 1$). Noticing that the graph has a logarithmic scale as the

x-axis, a small error in evaluating the Reynolds number can induce a large shift in experimental data. Indeed, Pfhaler et al. [8] observed large error bounds for the C^* -data in the smallest channels which may also imply (unreported) errors with respect to the observed Reynolds numbers. Fig. 5 indicates that the simulations reproduce the experimental data trend except for the developed

nolds numbers. Fig. 5 indicates that the simulations reproduce the experimental data trend except for the case of the $47.5 \times 10 \mu m$ channel, for which C^* appears to be independent of the Reynolds number. The dotted-line graphs represent the average effect of viscous dissipation on C^* , which were obtained using half the temperature rises in the channels.

3. Results and discussion

Viscous dissipation, influencing the fluid temperature and ultimately the friction factor in microconduits, was investigated for a simplified case (Section 3.1), geometric effects (Section 3.2), flow regime effects (Section 3.3), and fluid property effects (Sections 3.4 and 3.5). The impact of viscous dissipation for a basic convection heat transfer case is discussed in Section 3.6.

3.1. One-dimensional case

Revisiting Fig. 3, the results obtained from onedimensional and three-dimensional analyses are compared. The slopes are identical for both cases after the flow is fully-developed. The difference between the onedimensional approximation and the three-dimensional analysis is the entrance effect. The axial temperature gradient near the inlet of the conduit is higher than that of the exit. Hence, the simple and robust one-dimensional analysis can be used to investigate thermally fullydeveloped and low-Reynolds number cases where entrance effects are negligible.

3.2. Geometric effects

Conduit size and thermal boundary effects. Fig. 6 shows the effect of tube size on the fluid temperature changes between inlet and exit of the channels. The temperature rise is a strong function of pipe size. It is more significant for the adiabatic wall case. In the 20 μ m channel, the temperature rises by 20 K, resulting in a reduction of the dynamic viscosity by 33%. Assuming the Reynolds number at the inlet to be 2000, it increases to 2985 at the exit. This implies that the effect of the temperature change due to viscous dissipation should be taken into account for microchannels. For the constant-wall-temperature case, the fluid temperature rises by 4.5 K which reduces the dynamic viscosity of the fluid by 9% in the 20 μ m channel.

Aspect ratio effect. Fig. 7 shows the effect of channel aspect ratio on the temperature change in a channel of $D_{\rm h} = 6 \, \mu m$, which corresponds to the $110 \times 3.0 \, \mu m$ channel case of Pfhaler et al. [8]. As the aspect ratio deviates from unity, the effect of viscous dissipation increases. In rectangular channels, viscous dissipation increases the fluid temperature even for low Reynolds numbers (i.e., $Re_{D_h} \approx 0.2$).



Fig. 6. Tube size effect on temperature change, i.e., viscous dissipation (water, Re = 2,000): (a) adiabatic wall case and (b) constant wall temperature case.



Fig. 7. Effect of channel aspect ratio on temperature change due to viscous dissipation for iso-propanol flows in channels of $D_{\rm h} = 6 \,\mu {\rm m}$ with Re = 0.2.

3.3. Reynolds number effect

Investigating Eq. (7), the viscous dissipation term is proportional to the square of the Reynolds number. The effect of the Reynolds number on the temperature rise is shown in Fig. 8. The Reynolds number effect on viscous dissipation is stronger for the constant wall temperature case.

Using the property data of water at 300 K and assuming the $(T_m - T_w)$ -effect to be $\mathcal{O}(1)$, the relative magnitude of each coefficient in Eq. (7) has been calculated for Re = 20, 200, 2000 (see Table 2). For higher



Fig. 8. Reynolds number effect on mean temperature rise (water, $D_{\rm h} = 74 \ \mu m$).

Agnitude comparisons between convective, and dissipation terms in tubes									
$R [m] \qquad Re = 20$			Re = 200		Re = 2000				
	CONV* DISS	DIFF DISS	CONV DISS	DIFF DISS	CONV DISS	DIFF DISS			
10^{-3}	$3.45 \times 10^{+4}$	$5.81 \times 10^{+5}$	$3.45 \times 10^{+3}$	$5.81 \times 10^{+3}$	$3.45 \times 10^{+2}$	5.81×10			
10^{-4}	$3.45 \times 10^{+2}$	$5.81 \times 10^{+3}$	$3.45 \times 10^{+1}$	$5.81 \times 10^{+1}$	$3.45 \times 10^{+0}$	5.81×10^{-1}			
10^{-5}	$3.45 \times 10^{+0}$	$5.81 \times 10^{+1}$	3.45×10^{-1}	5.81×10^{-1}	3.45×10^{-2}	5.81×10^{-3}			
10^{-6}	3.45×10^{-2}	5.81×10^{-1}	3.45×10^{-3}	5.81×10^{-3}	3.45×10^{-4}	5.81×10^{-5}			

Table 2 Ν

CONV \triangleq convection term; DIFF \triangleq diffusion term; DISS \triangleq dissipation term.

Reynolds numbers, the relative magnitude of the dissipation term increases drastically with a decrease in channel size.

3.4. Varying viscosity effect

The temperature effect on viscous dissipation was observed by investigating the last term in Eq. (7). Expanding the Reynolds number using the definition, $Re_{\rm D} = \frac{\rho U_0 2R}{\mu}$, the last term is proportional to $\frac{\mu U_0^2}{R^2}$, where the dynamic viscosity of water changes with temperature, according to [16], as

$$\ln\left(\frac{\mu[kg/(m \cdot s)]}{1.788E - 3}\right) \approx -1.704 - 5.306 \left(\frac{273}{T}\right) + 7.003 \left(\frac{273}{T}\right)^2$$
(10)

For iso-propanol, a relation from [17] was used for 293 < T < 333 K

 $\mu(T) \approx 4.266 \times 10^{-7} T^2 - 3.016 \times 10^{-4} T + 5.398 \times 10^{-2}$ (11)

As mentioned, the dynamic viscosity of water reduces by about 20% for a temperature increase of 10 K, which implies that the viscous dissipation effect decreases downstream of the channel. With reference to Fig. 6, curves depicted with symbols represent mean fluid temperature changes under the assumption of constant viscosity, i.e., μ is evaluated at $\mu(T_0)$, while the line curves are the fluid temperatures taking viscosity changes into account, i.e., $\mu = \mu(T)$. Clearly, the variable viscosity effect is strongest for the smallest microtube subject to the adiabatic wall condition. Numerically, with respect to the inlet temperature of 300 K, the exit temperature decreased by 24% in the 20 µm, 9% in the 40 µm, and 3% in the 74 µm tube (Fig. 6a). The temperature changes are 8% in the 20 μ m, 2% in the 40 μ m, and 0.8% in the 74 µm tube for the constant temperature wall case (Fig. 6b).



Fig. 9. Comparison of heat source term impact due to viscous dissipation, S_v , and thermal boundary, S_h , for $100 \le Re \le 2000$: (a) in a 74 µm square channel and (b) Fractions of source terms in 40 and 74 µm square channels.

3168

3.5. Fluid properties effect

In order to investigate the effect of viscous dissipation on the temperature rise in a tube, the temperature distributions in a hydraulically fully-developed flow field was simulated for water, methanol and iso-propanol (see Table 1). Revisiting Fig. 3, the effect of working fluid, and hence implicitly the Eckert number (Ec = Br/Pr), on the axial temperature profile can be seen. Because the Eckert number is inversely proportional to the specific heat capacity of the working fluid, the mean temperatures of methanol and iso-propanol increase more, due to viscous dissipation, than the water temperature. For example, when comparing methanol and iso-propanol for the same Eckert number, the temperature rise for iso-propanol would be higher because of its higher dynamic viscosity and lower heat capacity; as a result, the average velocity would be higher for a given Reynolds number.

3.6. Effect on heat transfer

It has been shown that significant temperature increases may occur due to viscous dissipation even for a fluid with high viscosity and low heat capacity, subjected to very low Reynolds number flow. For example, the smallest microchannel of Pfhaler et al. [8], when assuming the inlet coolant temperature to be 20 °C, the exit temperature was $T_{\text{exit}} \approx 50$ °C. It is evident, from Eq. (3a), that a heated boundary can be regarded as a source term as viscous dissipation term works, as a heat source inside the domain. Adopting the 74 µm square channel of Judy et al. [7], the strengths of the source terms due to viscous dissipation and a thermal boundary condition are compared (Fig. 9). The viscous dissipation source term increases with the Reynolds number, while the heat transfer source term is kept constant. The fraction of viscous dissipation decreases with the heat transfer rate. In a smaller channel, the heat transfer source term will decrease linearly with channel width, while viscous dissipation will increase inversely proportional to the quadruple of the channel width size. Hence, the fraction of the heat dissipation source term increases rapidly.

4. Conclusions

The following conclusions can be drawn from this validated computer simulation study:

• By comparing the magnitude of each term in the governing heat transfer equation, the viscous dissipation effect on the friction factor was found to increase as the system size decreases. Specifically, for water flow in a tube with $D < 50 \mu m$, viscous dissipation becomes significant and hence should be taken into

consideration for all experimental and computational analyses.

- Channel size, the Reynolds number and the Brinkman number (or the Eckert number and the Prandtl number) are the key factors which determine the impact of viscous dissipation. Viscous dissipation effects may be very important for fluids with low specific heat capacities and high viscosities, even in relatively low Reynolds number flows.
- The effect of viscosity change, caused by variations in fluid temperature, on viscous dissipation was found to be measurable for flows in a long channel with a small hydraulic diameter. For liquids, the viscous dissipation effect decreases as the fluid temperature increases.
- The aspect ratio of a channel, i.e., height vs. width, plays an important role in viscous dissipation. Specifically, as the aspect ratio deviates from unity, the viscous dissipation effect increases.
- Viscous dissipation increases rapidly with a decrease in channel size and hence should be considered along with imposed boundary heat sources.
- Ignoring the viscous dissipation effect could ultimately affect friction factor measurements for flows in microconduits.

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